1.1: Linear Waves

For simplicity, assume only one space dimension, and that a typical field variable is $u(x,t)$. Linear waves can then be represented by the Fourier component,

$$u = Real[A \exp(ikx - i\omega t)], \tag{1}$$

where k is the *wavenumber*, ω is the wave *frequency* and A is the wave amplitude, which may also be a function of k. The full solution is obtained by a superposition of such components.

The wave dynamics are determined by the dispersion relation

$$\omega = \omega(k), \tag{2}$$

whose precise form is determined by the physical system under consideration. For instance, for water waves,

$$\omega^2 = gk \tanh(kh), \tag{3}$$

where h is the still water depth, and g is gravity. Here there are two branches of the dispersion relation.